On the Identity of Fermat's equation

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Abstract:

In order to develop a simple and short analytical proof of Fermat's last theorem based on the identity $x+y-z$ where x,y and z are independent variables of the equation corresponding to Fermat's last theorem, we have examined the most suitable property of $x+y-z$ for our purpose.

In this paper it is shown that $x+y-z$ and x^2+xy+y^2, which is co-prime to x,y,z have factor in common in case of the prime number five, and in case of the prime number seven $x+y-z$ and $(x^2+xy+y^2)^2$ have similar factor in common.

1.Introduction

Fermat's Last Theorem, here after referred to as FLT, known as the most difficult and famous theorem in Mathematics[1] and it has been first proved by Andrew Wiles using advanced mathematics. It is well known that this proof is very lengthy and difficult to understand. We are interested in producing a simple proof of FLT like in[2] , and we believe that this can be achieved by using the property of $x+y-z$ which we call the identity of the equation corresponding to FLT.

2.Methodology and proof

In the proof of our result $x+y-z$ and x^2+xy+y^2 have a common factor , we use the Werebrusow Identity [4], and that x derive our result using elementary mathematics. It should be noted that t is a factor common to $x+y-z$ and x^2+xy+y^2 in case of $p=5$ but when $p=7$ it is common to $x+y-z$ and $(x^2+y^2+xy)^2$ as we will prove in the following.

FLT can be stated as that the equation

$z^n = y^n \quad x^n, (x, y \; =$

(1)

which we have called Fermat's equation ,has no non-trivial integer solutions for x, y, z, where n is a positive (> 2)integer. The Identity $x + y - z$ is given [4] by Werebrusow Identity

$$(x + y - z)^p = \frac{4p}{2^p}(x + y)(z - x)(z - y) \times \sum_{i+j+k=\frac{(p-3)}{2}} (p - 1)! \frac{(x + y)^{2i}(z - y)^{2j}(z - x)^{2k}}{(2i + 1)!\,(2j + 1)!\,(2k + 1)!} \quad (2)$$

$i, j, k \geq 0$

where $p(\geq 3)$ is odd.

In case of $= 3$, the smallest odd prime, it easy to show[4] that

$(x + y - z)^3 = 3(x + y)(z - x)(z - y)$

(3)

We use the above identity to show that

$$(x + y - z)^5 = 5(x + y)(z - x)(z - y)k^5$$

$k^5 = \frac{1}{2}\,[\,(z - x)^2 + (z - y)^2 + (x + y)^2\,]$

(4)

Therefore $(x + y - z) = 5^m ughk$, where we have assumed that $y = 5^m ue$ where $e$is an integer co-prime to $5u$ and $m \geq 2$according to a theorem of Germain Sophie ,and k is co-prime withx, y, z.Also, the Barlow relationgives$(z - x) = 5^{5m-1}u^5$, $(z - y) = h^5, (x + y) = g^5$.

Now let us find the factors of k, where$(x + y - z) = 5^m ughk$, if there is any.

$$t^7 = \frac{7^6}{2^5}[6(x+y)^4 + 6(z-x)^4 + 6(z-y)^4 + 20(z-x)^2(z-y)^2 + 20(z-x)^2(x+y)^2 + 20(z-y)^2(x+y)^2]$$

Let us consider the term

$$[6[(x+y)^2 + (z-x)^2 + (z-y)^2]^2 + 8(z-x)^2(z-y)^2 + 8(x+y)^2(z-x)^2 + 8(z-y)^2(x+y)^2]$$

This can be expressed as

$$24[-z(x+y-z) + x^2 + y^2 + xy]^2 + 8z^2(x+y-z)^2) - 16zxy(x+y-z) - 16(x+y)^2[z(x+y-z)(x+y)^2 + (x^2+y^2+xy)^2$$

It is clear that t is a factor of $(x^2+y^2+xy)^2$.

Now, let us consider Class II solutions

In case of Class II solutions, Barlow relations give

$(x+y) = g^7, (z-x) = 7^{7m-1}u^7, (z-y) = h^7$ if we assume

$y = 7^m ue$ and $(7^m u, e) = 1$.Then t is cp-prime to $x, y, (x+y)$ and t is a factor of $(x^2+y^2+xy)^2$ as in case of Class I solutions.

We conjecture that for all odd primes the above result is true concerning x^2+y^2+xy and $(x^2+y^2+xy)^2$ and the Identity.

3. Summary and conclusions

We have shown that the Identity known as $x+y-z$ of the equation corresponding to Fermat's last theorem and the term x^2+xy+y^2 has factor, co-prime to $x, y, z,$ in common when $p = 5$.But in case of $p = 7$ the identity and $(x^2+y^2+xy)^2$ have a factor in common. We conclude that this property is quite general. In other words, if $p(> 3)$ is a prime and $2p+1$ is also a prime , then $x+y-z$ and x^2+xy+y^2 have a factor in common. If $2p+1$ is not a prime, then $x+y-z$ and $(x^2+y^2+xy)^2$ have a factor in common.

How the identity can be used to prove Fermat's last theorem for all indices, which is designed by one of us, will be published in the near future.

References

[1] Edwards H.M Fermat's last theorem, A Genetic Introduction to Algebraic Number, Theory, *Springer -Verlag*, 1977.

[2]Piyadasa R.A.D., (2011)March A Simple and short analytical proof of Fermat's last theorem,CMNSEM,Vol.2,No.3,.57-63

[3] Rihenboin P.(1991)Fermat's Last Theorem for amateurs, SpringerVerlag, New York