JOURNAL OF MULTIDISCIPLINARY & TRANSLATIONAL RESEARCH

Journal of Multidisciplinary and Translational Research (JMTR)

journal homepage: https://journals.kln.ac.lk/jmtr/

Cheeger constant of some families of graphs

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Abstract

The Cheeger constant, introduced by Jeff Cheeger in 1970, plays a vital role in understanding the connectivity and bottleneck properties of a graph. It serves as a fundamental concept in graph theory and network analysis, especially in partitioning problems and clustering applications in data science, machine learning, and related fields. A large Cheeger constant indicates strong connectivity, while a small value reflects the presence of sparse cuts within the graph structure. The Cheeger constant of some families of graphs, including 2-comb graphs, complete graphs, cubic graphs and quantum graphs such as cycle graphs, butterfly graphs, flower graphs and symmetric flower dumbbell graphs has been studied in the literature. This paper focuses on determining the Cheeger constant of various families of graphs, including path graphs, ladder graphs, Cartesian products of paths with cycles, and roach graphs. Even though finding Cheeger constant for a large graph is non-deterministic-polynomial time-hard hard (NP) problem, in this research, we derive closed-form formulas for the Cheeger constant using an optimum number of subsets, while varying the number of vertices in the graph. All the considered graphs share a similar structural pattern, with path graphs serving as their subgraphs, which allows for a unified approach in analyzing their Cheeger constants. Calculating the Cheeger constant for these families can help better understand their connectivity and separability. The findings are particularly valuable in applications such as spectral graph theory, network design, and data clustering. This study also highlights the usability of Cheeger constant for analyzing diverse graph families, bridging theoretical insights with practical applications in network partitioning and beyond.

Keywords: Cheeger constant, Roach graph, Ladder graph, Graph partition

Article info

Article history:
Received 23rd February 2026
Received in revised form 10th March 2026
Accepted 25th April 2026
Available online 17th April 2026

ISSN (E-Copy): ISSN 3051-5262
ISSN (Hard copy): ISSN 3051-5602
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Introduction

A graph cut is a partitioning of a vertex set of a graph into two or more disjoint sets. It is a fundamental concept used in areas such as computer vision, image segmentation, network design, and machine learning. Recent studies showed that Cheeger cut is performing well in partitioning problems (Zhu, W. 2025). The Cheeger constant is a fundamental concept in both Riemannian geometry and graph theory, serving as a quantitative measure of how easily a space or network can be partitioned into disjoint subsets (Chavel, 1984). The significance of Cheeger constant appears in spectral properties of associated operators, such as the Laplace-Beltrami operator in continuous settings and the Laplacian matrix in discrete graphs (Cheeger, J. 1970).

In computer networking, a higher Cheeger constant indicates robust connectivity, suggesting that the network can bear node or connection failures without fragmenting. This insight guides the design of resilient communication infrastructures. In machine learning and data analysis, the Cheeger constant supports spectral clustering by providing a measure of the quality of partitions or clusters within the data, ensuring that clusters are well separated. In social networks, it can detect communities with dense internal interactions and sparse external links, while in image segmentation it helps to isolate regions by minimizing boundary cuts relative to region size. Moreover, the Cheeger constant is helpful in modelling disease spread in contact networks, where a larger Cheeger constant indicates higher connectivity, leading to faster transmission of infections across the network. Chang, K.C. has analyzed both theory and algorithms for the Cheeger cut based on the graph 1-Laplacian (Chang, K.C., 2015). Fan Chung, in his research, discusses the relationship between the Laplacian of graphs and Cheeger inequalities, providing insights into how spectral properties of graphs relate to their expansion properties (Chung, F.R.K., 1997). The Cheeger constant for the family of distance-regular graphs has been studied (Qiao, Z. *et al*., 2020). Formulas for the Cheeger constant of some families of graphs have been derived, including cycle graphs, 2-comb graphs, complete graphs, and cubic graphs (Opio, S. *et al*., 2019). Some relations between the Cheeger constant and connectivity of finite digraphs has also been derived (Oshikiri, 2002). The Cheeger constant of quantum graphs such as cycle graphs, butterfly graphs, flower graphs and symmetric flower dumbbell graphs has been studied in Kennedy *et al.,* 2016. This study focuses on finding Cheeger constant of some families of graphs, including paths, ladder graphs, roach graphs and cartesian products of paths with cycles since they provide insights into connectivity and expansion of graph properties.

Methodology

The theoretical concepts used in this study were explained in the preliminaries section. A graph was denoted by $G = (V, E)$, consisting of a set of vertices $V = \{v_1, v_2, \dots, v_n\}$ and a set of edges $E = \{e_1, e_2, \dots, e_m\}$, where $e_i(1 \leq i \leq m)$ represented an edge connecting the vertices v_i and v_j. Simple undirected graphs were considered in this study.

Preliminaries

Ladder Graph: Ladder graph is denoted by L_{2k} $(k \geq 2, k \in \mathbb{Z}^+)$, where the number of vertices is $2k$ and the number of edges is $3k - 2$. This graph resembles the shape of a ladder, with two parallel paths joined by rungs, as shown in Figure 1. Mathematically, ladder graph with vertex set V and edge set E was defined as follows:

$V_1 = \{x_i | 1 \leq i \leq k\}$, $V_2 = \{y_i | 1 \leq i \leq k\}$, where V_1, V_2 were disjoint subsets of set V.

$E_1 = \{(x_i, x_{i+1}) | 1 \leq i \leq k-1\}$, $E_2 = \{(y_i, y_{i+1}) | 1 \leq i \leq k-1\}$, $E_3 = \{(x_i, y_i) | 1 \leq i \leq k\}$, where E_1, E_2, and E_3 were disjoint subsets of edge set E.

Here $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E_3$.

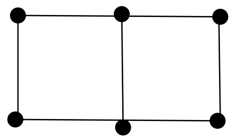

Figure 1. Ladder graph L_6, $(k = 3)$

Roach Graph: Roach graph resembled the shape of cockroach with body and two antennas. This graph was denoted by $R_{n,k}$ $(n \geq 1, k \geq 2)$ and it was a bounded degree planer graph with a vertex set $V = V_1 \cup V_2$ and an edge set $E_1 \cup E_2 \cup E_3$ as defined below:

$V_1 = \{x_i | 1 \leq i \leq n+k\}$, $V_2 = \{y_i | 1 \leq i \leq n+k\}$

$E_1 = \{(x_i, x_{i+1}) | 1 \leq i \leq n+k-1\}$, $E_2 = \{(y_i, y_{i+1}) | n+k+1 \leq i \leq 2(n+k)-1\}$,

$E_3 = \{(x_i, y_i) | n+1 \leq i \leq n+k\}$

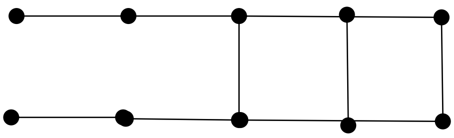

Figure 2. Roach graph $R_{2,3}$

The volume of a graph G was denoted by $vol(G) = \sum_{i=1}^{|V(G)|} d_i$, was the sum of the degrees of vertices in $V(G)$. The volume of a subset $A \subseteq V(G)$ was denoted by $vol(A) = \sum_{i \in A} d_i$.

Cartesian product of cycle graphs with path graphs

In this study, cycle graph C_m was considered with vertex set $V = \{c_i | 1 \leq i \leq m\}$ and edge set $E = \{(c_i, c_{i+1}) | 1 \leq i < m\} \cup (c_1, c_m)\}$. Similarly, the path graph P_n was considered with vertex set $V = \{p_i | 1 \leq i \leq n\}$ and edge set $E = \{(p_i, p_{i+1}) | 1 \leq i < n\}$. The Cartesian product of $C_m \times P_n$ consisted of n copies of cycles, each corresponding to a vertex of the path graph, as given in Figure 3.

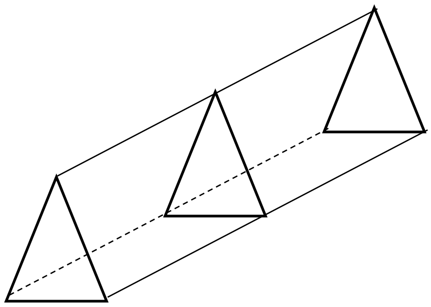

Figure 3. Cartesian product of cycle graph with path graph ($C_3 \times P_3$)

Graph cut: A subset of edges that disconnects the graph was called a graph cut. Let G be a weighted graph with a weighted adjacency matrix given by $W = (w_{ij})$. For a given non-empty subset $A, B \subset V$ and $A \cap B = \emptyset$, the graph cut was denoted by $cut(A,B) = \sum_{i\in A, j\in B} w_{ij}$. For unweighted graphs, the graph cut was defined as the number of cutting edges.

Cheeger constant-edge expansion:

Let $G = (V, E)$ be a graph. For a non-empty subset $S \subset V(G)$, $h_G(S)$ was defined as

$h_G(S) = \frac{cut(S, V\backslash S)}{\min(vol(S), vol(V\backslash S))}$, where $V\backslash S$ denoted the set of vertices not belonging to V. The Cheeger constant was then defined as $h_G = \min_S h_G(S)$.

Cheeger constant of some graph families:

The Cheeger constant of different families of graphs was discussed under the following theorems:

Theorem 1: The Cheeger constant of a path graph $P_n(n \geq 2, n \in \mathbb{Z}^+)$ was given by (Perera, *et, al.*, 2012)

$$h_{P_n} = \begin{cases} \frac{1}{n-1}, & n \;\; is \;\; even \\ \frac{1}{n-2}, & n \;\; is \;\; odd \end{cases}$$

(Proof): The vertex set was partitioned into two subsets A and $V\backslash A$ with exactly one cutting edges between them. It was assumed that $|A| \leq |V\backslash A|$. Then $|V\backslash A| \geq \left\lceil \frac{n}{2} \right\rceil$ and

$$h_G = \min \frac{cut(A, V\backslash A)}{\min(vol(A), vol(V\backslash A)} = \frac{1}{\min\left(2\left\lfloor\frac{n}{2}\right\rfloor - 1, 2\left\lceil\frac{n}{2}\right\rceil - 1\right)} = \frac{1}{2\left\lfloor\frac{n}{2}\right\rfloor - 1}$$

For n even, $h_{P_n} = \frac{1}{n-1}$ and for n odd, $h_{P_n} = \frac{1}{n-2}$.

Theorem 2: The Cheeger constant of the ladder graph was given by $L_{2k}=\begin{cases}\frac{2}{6k-6\alpha-2}, & k \le 2\alpha \\ \frac{2}{6\alpha-2}, & k \ge 2\alpha\end{cases}$, where $(k \ge 2, 1 \le \alpha < k, \alpha, k \in \mathbb{Z}^+)$.

The Cheeger constant was computed by considering different partitions of the vertex set.

Case (i) Suppose that the graph was partitioned by a horizontal cut. The set of vertices belonging to the upper path of the graph was denoted by $A_1 \subseteq V(G)$. Then

$A_1 = \{x_i | 1 \le i \le k\}$ and $V \backslash A_1 = \{y_i | 1 \le i \le k\}$

$$h_G(A_1) = \frac{k}{\min(3k-2, 3k-2)} = \frac{k}{3k-2} = \frac{2k}{vol(L_{2k})}$$

Case (ii) Suppose the graph was partitioned vertically. In this case, $A_2 \subseteq V(G)$ denoted the set of vertices lying on the left side of the graph, including vertices from both the upper and lower paths.

Let $A_2 = \{x_i | 1 \le i \le \alpha\} \cup \{y_i | 1 \le i \le \alpha\}$ where $1 \le \alpha < k$ and $V \backslash A_2 = \{x_i | \alpha+1 \le i \le k\} \cup \{y_i | \alpha+1 \le i \le k\}$. The Cheeger ratio was computed as

$$h_G(A_2) = \frac{2}{\min\left(2(3\alpha-1), 2(3k-3\alpha-1)\right)}.$$

$\min\left(2(3\alpha-1), 2(3k-3\alpha-1)\right) = 6k-6\alpha-2 \;\; if \;\; k < 2\alpha$

and $\min\left(2(3\alpha-1), 2(3k-3\alpha-1)\right) = 6\alpha-2 \;\; if \;\; k > 2\alpha$.

Equality occurred when $k = 2\alpha$. Then

$$h_G(A_2) = \begin{cases}\dfrac{2}{6k-6\alpha-2}, & k \le 2\alpha \\ \dfrac{2}{6\alpha-2}, & k \ge 2\alpha\end{cases}$$

Considering Case (i), and Case (ii), Case(ii) yielded the minimum Cheeger constant.

Case (iii) Suppose the graph was partitioned both horizontally and vertically. $A_3 \subseteq V(G)$ denoted the set of vertices lying on one side of the graph. Then

$A_3 = \{x_i | 1 \le i \le k\} \cup \{y_i | 1 \le i \le \alpha\}$ and $A_3 = \{y_i | \alpha+1 \le i \le k\}$.

$h_G(A_3) = \frac{k-\alpha+1}{\min(3k+3\alpha-3, \; 3k-3\alpha-1)} = \frac{k-\alpha+1}{3k-3\alpha-1}$, for $\alpha \ge 1$

Comparison of Case (iii) with Case (ii), showed Case(ii) yielded the minimum.

Considering all the cases above, Cheeger constant of L_{2k} was given by $\begin{cases}\frac{2}{6k-6\alpha-2}, & k \le 2\alpha \\ \frac{2}{6\alpha-2}, & k \ge 2\alpha\end{cases}$.

Theorem 3: The Cheeger constant of the roach graph $R_{2k,k}$ is given by $\frac{1}{4k-1}$, where $n = 2k. k \geq 2$

(Proof) The proof was divided into five cases. In each case, a partition of the vertex set $V(G)$ into two disjoint subsets was considered. Let $A_i \subseteq V(G)$, for $i = 1,2,3,4,5$, denote the subset corresponding to Case i. The five cases were analysed as follows:

Case (i) $A_1 = \{x_i | 1 \leq i \leq 2k\}$ and $V \backslash A_1 = \{y_i | 1 \leq i \leq 3k\} \cup \{x_i | 2k+1 \leq i \leq 3k\}$

$$h_G(A_1) = \frac{1}{\min(4k-1, 10k-3)} = \frac{1}{4k-1}.$$

Case (ii) $A_2 = \{x_i | 1 \leq i \leq 2k+\alpha\} \cup \{y_i | 1 \leq i \leq 2k+\alpha\}$, where $0 \leq \alpha < k$ and

$$V \backslash A_2 = \{x_i | 2k+\alpha+1 \leq i \leq 3k\} \cup \{y_i | 2k+\alpha+1 \leq i \leq 3k\}.$$

$$h_G(A_2) = \frac{2}{\min(8k-2+6\alpha,\ 6k-6\alpha-2)} = \frac{2}{6k-6\alpha-2}.$$

Considering Case (i), Case (ii),

$$h_G = \min\left\{\frac{1}{4k-1}, \frac{1}{3k-3\alpha-1}\right\} = \frac{1}{4k-1}, \text{ since } \frac{1}{4k-1} - \frac{1}{3k-3\alpha-1} = \frac{-(k+3\alpha)}{(4k-1)(3(k-\alpha))-1} < 0.$$

Case (iii) $A_3 = \{x_i | 1 \leq i \leq 2k\} \cup \{y_i | 1 \leq i \leq 2k\}$ and

$$V \backslash A_3 = \{x_i | 2k+1 \leq i \leq 3k\} \cup \{y_i | 2k+1 \leq i \leq 3k\}.$$

$$h_G(A_3) = \frac{2}{\min(2(4k-1), 2(3k-1))} = \frac{2}{6k-2}.$$

Comparison of Case (iii) with Case (i), showed that Case(i) yielded the minimum.

Case (iv) Let $A_4 = \{x_i | 1 \leq i \leq 3k\}$ and $V \backslash A_4 = \{y_i | 1 \leq i \leq 3k\}$. Then $vol(A_4) = vol(V \backslash A_4) = 7k-2$ and $h_G(A_4) = \frac{k}{7k-2}$. Comparing with Case(i), Case(i) yielded the minimum.

Case(v) Let $A_5 = \{x_i | 1 \leq i \leq \alpha\} \cup \{y_i | 1 \leq i \leq \alpha\}$, where $1 \leq \alpha < 2k$ and $V \backslash A_5 = \{x_i | \alpha+1 \leq i \leq 3k\} \cup \{y_i | \alpha+1 \leq i \leq 3k\}$.

$$h_G(A_5) = \frac{2}{\min(4\alpha-2, 14k-4\alpha-2)}$$

$$\min(4\alpha-2, 14k-4\alpha-2) = \begin{cases} 14k-4\alpha-2 & \frac{7k}{4} \leq \alpha \leq 2k, \\ 4\alpha-2 & otherwise \end{cases} \text{ and}$$

$$h_G(A_5) = \begin{cases} \frac{2}{14k-4\alpha-2} & \frac{7k}{4} \leq \alpha \leq 2k, \\ \frac{2}{4\alpha-2} & otherwise \end{cases}$$

Comparison of Case (v) with Case (i):

$min\left(\frac{1}{2\alpha-1}, \frac{1}{4k-1}\right) = \frac{1}{4k-1}$ and $min\left(\frac{1}{7k-2\alpha-1}, \frac{1}{4k-1}\right) = \frac{1}{4k-1}$. Case (i) yielded the minimum.

Considering all the above cases, Cheeger constant of $R_{2k,k}$ is $\frac{1}{4k-1}$.

Cartesian product of cycles and paths represented cylindrical grid structures, which appeared in computer networks, transportation systems, and communication models. Finding the Cheeger constant was helpful in understanding the separability of grid-like structures. The next Theorem gave the formula for Cheeger constant of the Cartesian product of cycles and paths.

Theorem 4: The Cheeger constant of the Cartesian product of cycle graphs C_m $(m \geq 3)$ with path graphs P_n $(n \geq 2)$ is

$$h_G(C_m \times P_n) = \begin{cases} \dfrac{m}{\left\lfloor \frac{n}{2} \right\rfloor 4m - m}, & m \leq 2n \\ \dfrac{n}{\left\lfloor \frac{m}{2} \right\rfloor (2n-1)}, & m > 2n \end{cases}$$

(Proof) Let $G = C_m \times P_n, (n \geq 2, m \geq 3, n, m \in \mathbb{Z}^+)$ be a graph consisting of n copies of the cycle C_m each corresponding to a vertex of the path graph P_n.

Case (i) Suppose the graph was partitioned vertically. Let $A_1 \subseteq V(G)$ denote the set of vertices on one side of the partition, including vertices from both the path and the cycles on that side. Then $A_1 = \{(c_i, p_j) \mid 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor, 1 \leq i \leq m\}$ and $V \backslash A_1 = \{(c_i, p_j) \mid \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq j \leq n, 1 \leq i \leq m\}$.
Then $vol(A_1) = \left\lfloor \frac{n}{2} \right\rfloor (vol(C_m) + 2m) - m, vol(V \backslash A_1) = \left\lceil \frac{n}{2} \right\rceil (vol(C_m) + 2m) - m$ and $cut(A_1, V \backslash A_1) = m$.
Hence, $h_G(A_1) = \frac{m}{\min\left(\left\lfloor \frac{n}{2} \right\rfloor 4m - m, \left\lceil \frac{n}{2} \right\rceil 4m - m \right)} = \frac{m}{\left\lfloor \frac{n}{2} \right\rfloor 4m - m}$
When n is even, $h_G(A_1) = \frac{1}{2n-1}$ and when n is odd, $h_G(A_1) = \frac{1}{2n-3}$.

Case (ii) Suppose the graph was partitioned horizontally along the cycles. The set of vertices on one side of the partition was denoted by $A_2 \subseteq V(G)$.
Let $A_2 = \{(c_i, p_j) \mid 1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor, 1 \leq j \leq n\}$ and $V \backslash A_2 = \{(c_i, p_j) \mid \left\lfloor \frac{m}{2} \right\rfloor + 1 \leq i \leq m, 1 \leq j \leq n\}$.
Then $vol(A_2) = n\ vol\left(C_{\left\lfloor \frac{m}{2} \right\rfloor}\right) + 2\left\lfloor \frac{m}{2} \right\rfloor (n-1) = 2(2n-1)\left\lfloor \frac{m}{2} \right\rfloor$, and
$vol(V \backslash A_2) = 2(2n-1)\left\lceil \frac{m}{2} \right\rceil$ and $cut(A_2, V \backslash A_2) = 2n$.
Therefore $h_G(A_2) = \frac{2n}{\min\left(\left\lfloor \frac{m}{2} \right\rfloor 2(2n-1),\ \left\lceil \frac{m}{2} \right\rceil 2(2n-1) \right)} = \frac{n}{\left\lfloor \frac{m}{2} \right\rfloor (2n-1)}$
When m is even, $h_G(A_2) = \frac{2n}{m(2n-1)}$ and when m is odd, $h_G(A_2) = \frac{2n}{(m-1)(2n-1)}$.

Now, Case (i) was compared with Case (ii):

If n is even and m is even then, $\frac{1}{2n-1} - \frac{2n}{m(2n-1)} = \frac{m-2n}{m(2n-1)} \geq 0$, if $m \geq n$
If n is even and m is odd then, $\frac{1}{2n-1} - \frac{2n}{(m-1)(2n-1)} = \frac{m-2n-1}{(m-1)(2n-1)} \geq 0$, if $m > 2n$
If n is odd and m is odd then, $\frac{1}{2n-3} - \frac{2n}{(m-1)(2n-1)} = \frac{(m-2n)(2n-1)+(2n+1)}{(m-1)(2n-1)(2n-3)} \geq 0$, if $m \geq 2n$
If n is odd and m is even then, $\frac{1}{2n-3} - \frac{2n}{m(2n-1)} = \frac{(m-2n)(2n-1)+4n}{m(2n-1)(2n-3)} \geq 0$, if $m \geq n$

Therefore, If m is even and $m \geq n$ or m is odd and $m > 2n$, then Case(ii) yielded the minimum, Otherwise, Case(i) yielded the minimum. This corresponded to a graph cut through the cycles.

Results

Cheeger constants of the families of graphs were calculated in the methodology section and listed below:

For the path graph P_n, $h_{P_n} = \begin{cases} \frac{1}{n-1}, & n \ is \ even \\ \frac{1}{n-2}, & n \ is \ odd \end{cases}$

For the ladder graph L_{2k}, $h_{L_{2k}} = \begin{cases} \frac{2}{6k-6\alpha-2}, & k \leq 2\alpha \\ \frac{2}{6\alpha-2}, & k \geq 2\alpha \end{cases}$, where $1 \leq \alpha < k$

For the roach graph $R_{2k,k}$, $h_{R_{2k,k}} = \frac{1}{4k-1}$.

For the Cartesian product of cycles and paths,

$$h_{C_m \times P_n} = \begin{cases} \dfrac{m}{\left\lceil \frac{n}{2} \right\rceil 4m - m}, & m \leq 2n \\ \dfrac{n}{\left\lceil \frac{m}{2} \right\rceil (2n-1)}, & m > 2n \end{cases}$$

Numerical illustrations

Tables 1, 2 and 3 summarize the Cheeger constants computed for path graphs, ladder graphs and roach graphs respectively, as they vary with the number of vertices.

Table 1: Cheeger constants of Path graphs

Number of vertices	10	20	30	55	60	211	300
Cheeger constant	0.11	0.0526	0.0348	0.0188	0.0169	0.0047 8	0.0033

Table 2: Cheeger constants of Ladder graphs L_{2k}

Number of vertices	10	20	30	60	200	300
Value of k	5	10	15	30	100	150
Value of α	2	5	8	15	50	75
Cheeger constant	0.2	0.071	0.05	0.0227	0.006711	0.004464

Table 3: Cheeger constants of Roach graphs $R_{2k,k}$

Number of vertices	30	60	180	600	900
Value of k	5	10	30	100	150
Cheeger constant	0.0526	0.02564	0.0084	0.00256	0.0016

Discussion

This study derived explicit formulas for the Cheeger constant of several graph families that were not previously characterized in the literature. The Cheeger constant is a fundamental measure of graph connectivity with deep connections to spectral graph theory and random walks on graphs (Chung, 1997; Mohar, 1989). Calculating Cheeger constant for a large graph is NP hard problem in general (Oehlers, M., Fabian, B., 2021). In this work, appropriate vertex subsets that enabled exact computation for path, ladder, roach, and cartesian product of cycles and path graphs were selected. The results showed that, as the number of vertices n increases, the Cheeger constant of each graph family decreases, indicating weaker connectivity in longer or larger graphs. It was observed that ladder graphs generally exhibit higher Cheeger constants for the same number of vertices, indicating stronger connectivity, whereas path graphs exhibited lower Cheeger constants, reflecting their linear structure and higher vulnerability to disconnection. According to the results obtained, Roach graphs typically have Cheeger constants higher than those of path and ladder graphs. This observation agrees with previous studies comparing expansion in linear graphs and highly connected structures (Chung 1997, Ding & Wang, 2025). Adding redundant edges to the graph increases edge expansion and enhances connectivity (Chung 1997). Overall, the findings strengthen the importance of structural properties in determining the connectivity and stability of complex networks.

Conclusion

The Cheeger constant is a fundamental concept in both Riemannian geometry and graph theory, which is used as a measure to indicate how a space or network can be partitioned into disjoint subsets by using the minimum number of edge cuts. Families of graphs considered in the study play a vital role in network structures, and Cheeger constant can be used to ensure the robustness of network structures against failures. In this study, a general formula is given for some graph families including path graphs, ladder graphs, roach graphs, and the cartesian product of cycles with paths. The study shows that the Cheeger constant decreases with increasing number of vertices of graphs, reflecting weaker connectivity in longer paths. Roach graphs exhibit higher Cheeger constants for the same number of vertices, indicating stronger connectivity and robustness, while ladder graphs have intermediate Cheeger constants.

Acknowledgement

The unwavering support extended by Shyama Dissanayaka for this work is sincerely remembered.

Declaration of Funding Sources

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Conflict of interest statement

The authors declare that there was no conflict of interest in conducting this research work.

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